Taxation and Long-Run Growth: Technical Appendix

by

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This appendix contains detailed descriptions of the computations and the analytics underlying the paper "Taxation and Long-Run Growth."

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1. Introduction

This appendix contains detailed documentation for the computer programs used to generate the results reported in "Taxation and Long-Run Growth." The model is described in the main text. Section 2 characterizes its equilibrium. Section 3 contains a detailed description of parameter choices. The model's steady state properties are compared with US data in section 4. Finally, section 5 provides documentation for individual program files and instructions on common procedures.

2. Characterization of Equilibrium

This section describes in detail the equilibrium conditions to be solved by the numerical procedure. It is assumed throughout, that the economy is on a balanced growth path; this allows to suppress time subscripts on all variables. The firm's problem is described in the main text and is not repeated here.

2.1 Baseline Case

2.1.1 Households

The baseline model has no altruistic bequests and no investment in the endowment h_1. Given the household problem as described in the main text, set up a Lagrangian

$$V(a_1,h_1)=\sum_{i=1}^{T}\beta^i\,u(c_i,l_i)+\lambda\left[\sum_{i=1}^{T}R_i^{-1}\{w_i\,h_i\,(1-l_i-v_i)+Z_i-c_i\,(1+\tau_c)-x_i\,p_i\}+a_1\right]$$
$$+\sum_{i=1}^{T}\varphi_i\{(1-\delta_h)\,h_i+G(i)-h_{i+1}\}+\sum_{i=1}^{T-1}\mu_i\,(1-l_i-v_i)$$

The constraints $l_i \geq 0$ never bind for the utility function chosen here. Similarly, Inada conditions imposed on $G(.)$ ensure that $v_i > 0$ and $x_i > 0$, except when $\varphi_i = 0$. The first order conditions are

$$\beta^i\,u_c(i)=(1+\tau_c)\,\lambda/R_i \quad (1)$$

$$\beta^i\,u_l(i)=w_i\,h_i\,\lambda/R_i+\mu_i$$

$$\varphi_{i-1}=(1-l_i-v_i)\,w_i\,\lambda/R_i+\varphi_i\{1-\delta_h+G_h(i)\};\ \varphi_T=0$$

$$\varphi_i\,G_v(i)=w_i\,h_i\,\lambda/R_i+\mu_i\text{, if }\varphi_i>0$$

$$\varphi_i \, G_x(i) = c x_i \, \lambda / R_i \text{, if } \varphi_i > 0$$

$$\mu_i \, (1 - l_i - v_i) = 0 \, ; \quad (1 - l_i - v_i) \geq 0$$

as well as the inequality constraints $\mu_i, \varphi_i, v_i, x_i \geq 0$, the law of motion for h and the present value budget constraint shown in the main text. Where $\varphi_i = 0$, replace the conditions for v and x by $v_i = x_i = 0$.

A solution to the household's problem consists of values $\{c_i, l_i, v_i, x_i, h_i, \mu_i, \varphi_i\}_{i=1}^{T}$ and λ that satisfy these first-order conditions, given prices $\{w_i, r_i, p_i\}_{i=1}^{T}$, tax rates $\tau_{c,i}$, and initial conditions h_1 and a_1. Asset holdings are determined residually from the flow budget constraint

(2) $$a_{i+1} = (1 + r_i) \, a_i + w_i \, h_i \, (1 - l_i - v_i) - x_i \, p_i - c_i \, (1 + \tau_{c,i}) + Z_i \, .$$

Inspection of the law of motion for φ reveals that the solution has two phases. At the end of life, there is a period where $\varphi_i = 0$, no investment in human capital takes place and no work is performed at any future point. Of course, this phase may consist only of date T. Once φ has turned positive, it remains positive at all earlier dates and human capital investment always takes place.

The prices faced by the household are determined as follows. Before taxes, the price of one unit of x is one. However, fraction Λ is paid for by lower earnings and therefore has unit cost $(1 - \tau_{l,i})$. The remainder is paid directly by the worker and costs $(1 - s_i - \tau_{l,i} \, \Gamma)$ per unit, where s is a subsidy rate and fraction Γ is tax deductible. The unit cost to the worker is thus $\Lambda \, (1 - \tau_{l,i}) + (1 - \Lambda) \, (1 - s_i - \tau_{l,i} \, \Gamma)$. Therefore,

$$p_i = 1 - (1 - \Lambda)(s_i + \tau_{l,i} \, \Gamma) - \tau_{l,i} \, \Lambda \, .$$

Firms pay a rental price of w^* per efficiency unit of labor services. The household pays a fraction τ_l in wage taxes. Before age T_1, $\tau_l = \tau_w$; thereafter, $\tau_l = \tau_w + \tau_o$. Thus,

$$w_i = (1 - \tau_{l,i}) \, w^* \, .$$

Finally, firms pay a rental price of r^* per unit of capital services, of which the household pays fraction τ_K in capital income taxes. In addition, the household loses fraction δ_K to depreciation, of which κ is tax deductible, such that depreciation costs $(1 - \kappa \, \tau_K) \, \delta_K$ per unit of capital. The after tax rate of return is therefore

$$r = (1 - \tau_K) \, r^* - (1 - \kappa \, \tau_K) \, \delta_K \, .$$

The cumulative discount factor is defined as $R_i = (1 + r)^i$.

2.1.2 Government

Government expenditures consist of government consumption $(\overline{G})$, transfers to households $(\overline{Z})$, and interest on previously issued debt. Revenues consist of tax payments on capital income, labor income and consumption. Consumption tax revenues are simply $T_c = \tau_c C$, where C is aggregate consumption. Capital income tax revenue, net of deductible depreciation, is the difference between rental payments by firms, $K(r^* - \delta_K)$, and receipts by households $K(r^*[1-\tau_K]-[1-\kappa\tau_K]\delta_K)$. Thus, $T_K = K(r^*\tau_K - \kappa\tau_K\,\delta_K)$. Wage tax revenues are the difference between labor incomes paid by firms, $w^* L$, and those received by households, including tax deductions and subsidies for investments in training:

$$\begin{aligned} T_L &= w^* L - w L - X(1-\overline{p}) \\ &= \sum_{i=1}^{T} \left\{ w^* h_i (1-l_i - v_i)\,\tau_{l,i} - x_i (1-p_i) \right\} (1+\gamma)^{1-i} (1+n)^{1-i} \end{aligned}$$

where X is aggregate investment in training and $\overline{p}$ is the average price paid by households for x; both are implicitly defined in the second equation.

The government's flow budget constraint on a balanced growth path is therefore

$$(1+\gamma)(1+n)D + T_c + T_K + T_L = (1+r)D + \overline{Z} + \overline{G} \ .$$

2.1.3 Aggregation

With balanced growth, total labor supply is determined by

$$L = \sum_{i=1}^{T} (1-l_i - v_i)\,h_i\,(1+\gamma)^{1-i}(1+n)^{1-i} \ .$$

Each household of age i contributes effective time $(1-l_i - v_i)h_i$. If the size of the cohort born at date 1 is normalized to one, cohort i contains $(1+n)^{1-i}$ members. The growth factor $(1+\gamma)^{1-i}$ accounts for the fact that older cohorts started out with smaller endowments, h_1.

Similarly, the aggregate capital stock must equal total household wealth minus government debt:

$$K = \sum_{i=1}^{T} a_{i,t}\,(1+\gamma)^{1-i}(1+n)^{1-i} - D \, .$$

Aggregate consumption can be written as

$$C = \sum_{i=1}^{T} c_i\,(1+\gamma)^{1-i}(1+n)^{1-i} \ .$$

Aggregate government transfers are

$$\overline{Z} = \sum_{i=1}^{T} Z_i \, (1+\gamma)^{1-i} \, (1+n)^{1-i} \ .$$

Finally, the growth rate of per capita output must equal that of the endowment h_1. Children are born $t_b - 1$ years after their parents with an endowment of εh_{t_h}; therefore,

$$(1+\gamma)^{t_b - 1} = \varepsilon h_{t_h} / h_1 \ . \qquad (3)$$

2.1.4 Balanced Growth Equilibrium

A balanced growth equilibrium consists of

- values for the household's choice variables $\{c_i, l_i, v_i, x_i, h_i, \mu_i, \varphi_i, a_i\}_{i=1}^{T}$ and λ
- prices: w_i, p_i, r, w^*, r^*
- aggregate quantities: Y, K, L
- policy variables: $\tau_K, \tau_w, \tau_c, \tau_o, \tau_{l,i}, s, \Lambda, \Gamma, \kappa, D/Y, G/Y$
- a per capita income growth rate, γ

that satisfy

- firm optimality conditions (given prices)
- household optimality conditions (given prices and initial conditions)
- aggregation conditions
- the government budget constraint.

How constant returns in G are achieved (spillover or ability effects) does not affect the equilibrium conditions.

2.2 Intergenerational Links

This section analyzes the changes to the balanced growth conditions that arise if parents are altruistically linked to their children in two ways: parents leave a financial bequest and they internalize that children "inherit" their human capital. Specifically assume that each parent has N_c children at age t_b who inherit a fraction ε of the parent's human capital at age t_h. The parent leaves a bequest of B_T to each child during his last period, which is received by children at age $t^* = T - t_b + 1$ (augmenting a_{t^*+1}). Let B_1 denote the bequest received by the parent, which he takes as given. Parents value the utility of each child with weight $b\beta^{t_b - 1}$ so that $b = 1$ corresponds to full altruism. The parent's problem is then to maximize the following Lagrangian:

$$V(B_1,h_1)=\sum_{i=1}^{T}\beta^i\,u(c_i,l_i)+b\,\beta^{t_b-1}\,N_c\,V(B_T,h_1^*)$$

(4) $$+\lambda\left[\sum_{i=1}^{T}R_i^{-1}\{w_i\,h_i\,(1-l_i-v_i)+Z_i-c_i\,(1+\tau_c)-x_i\;p_i\}+a_1+\frac{B_1}{R_{t*}}-\frac{N_c\,B_T}{R_T}\right]$$

$$+\sum_{i=1}^{T}\varphi_i\{(1-\delta_h)\,h_i+G(i)-h_{i+1}\}+\sum_{i=1}^{T-1}\mu_i\,(1-l_i-v_i)$$

where $h_1^*=\varepsilon\,h_{t_h}$. The only changes to the balanced growth conditions are two altered first-order conditions for the household. First, the first-order condition for h becomes

(5) $$\varphi_{i-1}=\varphi_i\,[1-\delta_h+G_h(i)]+\overline{\lambda}_i\;w_i\,(1-l_i-v_i)+I(i=t_h)\,\varepsilon\,N_c\,b\,\beta^{t_b-1}\,\frac{\partial V}{\partial h_1}(B_T,h_1^*)\,,$$

where $\overline{\lambda}_i\equiv\lambda/R_i$ and $I(i=t_h)$ is an indicator function that takes on the value 1 if i = t_h and 0 otherwise. The marginal value of h_1 is

$$\frac{\partial V}{\partial h_1}(B_1,h_1)=\varphi_0\equiv\varphi_1[1-\delta_h+G_h(1)]+\overline{\lambda}_1\,w_1\,(1-l_1-v_1)\,.$$

Note that φ_0 grows at the same rate as λ. Therefore, on a balanced growth path, the child's φ_0 is given by $\varphi_0\,(1+\gamma)^{t_b-1}$. It is therefore useful to define $\varphi_i^*\equiv\varphi_i/\overline{\lambda}_i$, which is time invariant. Since

$$\varphi_0^{ch}\,R_i/\lambda=(\varphi_0^{ch}/\lambda^{ch})\,R_i\,(\lambda^{ch}/\lambda)=\varphi_0^*\,R_i\,(1+\gamma)^{-\sigma(t_b-1)}\,,$$

equation (5) becomes

(6) $$\varphi_{i-1}^*(1+r_i)=\varphi_i^*[1-\delta_h+G_h(i)]+w_i\,(1-l_i-v_i)+I(i=t_h)\,\varepsilon\,N_c\,b\,\beta^{t_b-1}\,\varphi_0^*\,R_i\,(1+\gamma)^{-\sigma(t_b-1)}\,.$$

In the special case where children are born when the parent is aged T+1, t_h = T+1 and the same expression applies, of course with date T+1 values for all parental variables set to 0, i.e.,

$$\varphi_T^*=\varepsilon\,N_c\,b\,\beta^{t_b-1}\,\varphi_0^*\,R_T\,(1+\gamma)^{-\sigma(t_b-1)}\,.$$

Second, there is an additional first-order condition for the choice of the bequest left, B_T:

(7) $$\overline{\lambda}_T\;N_c=b\,\beta^{t_b-1}\,N_c\,\frac{\partial V}{\partial B}(B_T,h_1^*)\,,$$

where $$\frac{\partial V}{\partial B}(B_1,h_1)=\overline{\lambda}_{t*}$$

is the marginal value of a bequest received. Since from the first-order condition for c,

$$\beta^i\, u_c(i) = \overline{\lambda}_i\,(1+\tau_c)\,,$$

equation (7) simplifies to

$$b\,\beta^{t_b-1}\,\overline{\lambda}^{ch}_{t^*} = b\,\beta^{t_b-1+t^*}\,u_c^{ch}(t^*)/(1+\tau_c) = \beta^T\,u_c(T)/(1+\tau_c)\,,$$

where the superscript *ch* denotes variables pertaining to the child. Since $t^* = T - t_b + 1$, we have

(8) $$b\,u_c^{ch}(t^*) = u_c(T)\,.$$

The interpretation is straightforward. The parent must be indifferent between raising own consumption at age T by one unit and increasing consumption of one child (aged t^*) by one unit. On a balanced growth path, since the parent's parents were born (t_b – 1) periods before the parent: $B_1 = B_T\,[(1+n)(1+\gamma)]^{1-t_b}$.

2.2.1 Solving the Model With Bequests

If there is an operative bequest motive, growth rates and interest rates are linked by

(9) $$(1+\gamma)^\sigma = b^{1/(t_b-1)}\,\beta\,(1+r)\,.$$

This is a slight generalization of the well-known relationship for the infinite horizon case, which has $b = 1$. To see that (9) holds, note that

(10) $$u_c^{ch}(t^*) = [c_{t^*}(1+\gamma)^{t_b-1}]^{-\sigma}\,l_{t^*}^{\rho(1-\sigma)} = (1+\gamma)^{-\sigma(t_b-1)}\,u_c(t^*)\,.$$

Therefore, (8) implies $b\,(1+\gamma)^{-\sigma(t_b-1)}\,u_c(t^*) = u_c(T)$. But the Euler equation (1) implies

(11) $$u_c(T)\,/\,u_c(t^*) = (\beta^{t^*}\,R_{t^*})\,/(\beta^T\,R_T) = [\beta\,(1+r)]^{t^*-T}\,,$$

where $T - t^* = t_b - 1$ is the age difference between generations. Combining (10) and (11) implies (9).

This suggests the following solution algorithm. Iterate over guesses for the growth rate, γ. The implied interest rate is given by $1+r = b^{-1/(t_b-1)}(1+\gamma)^\sigma/\beta$. Since $r = r^*[1-\tau_K]-[1-\kappa\tau_K]\delta_K$ and the marginal product of capital is $r^* = \theta A k^{\theta-1}$, the implied capital-labor ratio is

(12) $$k^{\theta-1} = \frac{r+[1-\kappa\tau_K]\delta_K}{(1-\tau_K)\,\theta A}\,.$$

Solving the household problem for a given level of bequests received and left (both are linked by a balanced growth condition), yields new guesses for k and γ. However, the new guess for k is used to update the bequest level, which is reduced if k is higher than the value implied by (12). [This algorithm is not used in the current version of the programs.]

2.2.2 An Alternative Specification of Human Capital Linkages

Once it is assumed that parents internalize the intergenerational spillover, it is no longer useful to assume that the spillover occurs at one particular age because this implies a discontinuity in the incentives for human capital accumulation at that age. It is preferable to model the spillover as occurring over an extended period as the child grows up with the parent. In particular, assume a child learns a fraction $\varepsilon\, \nu_i$ during periods $I_1 \le i \le I_2$ it spends with the parent, where i is the parent's age. Then the human capital endowment of a new agent entering the model is

$$h_1^* = \varepsilon \sum_{i=I_1}^{I_2} \omega_i\, h_i \, .$$

This changes the balanced growth conditions as follows. The indicator function in (5) is replaced by ν_i for ages $I_1 \le i \le I_2$. The balanced growth condition (3) is replaced by

$$h_1^* = h_1\,(1+\gamma)^{t_b-1} = \varepsilon \sum_{i=I_1}^{I_2} \omega_i\, h_i \, .$$

To calibrate this, normalize $\omega_{I_1} = 1$ and assume $\omega_{I_2} = 0$ to preserve continuity of the parent's incentive for human capital investments. For simplicity also assume that ω_i is linear. Then $\omega_i = (I_2 - i)/(I_2 - I_1)$. Since ε can still be chosen to replicate the target growth rate, it only remains to choose the age range over which learning occurs. We assume that this begins when children are physically born (parental age 28, model age I_1 = 9) and ends when the child becomes an independent agent (parental age 47, model age I_2 = 28).

2.3 Investment in the Endowment

Instead of assuming that an agent's human capital endowment is "inherited" from a previous generation, assume that parents invest in their children's endowment. Specifically, assume that the child is "born" at physical age a_1^* with a human capital endowment of h_1^* of ε times the parent's human capital at age t_h. Until the child enters the model at age a_1, the parent can spend own time and goods to accumulate human capital for the child according to

$$h_{i+1}^* = (1-\delta_h^*)\, h_i^* + G^*(\tilde{v}_i^*\, h_i\, , x_i^*\, , \bar{h}_i)\, .$$

The notation for inputs is analogous to that for investment in own human capital. Note that it is the *parent's* human capital that determines the child's human capital accumulation. The child then enters the model with $h_1 = h^*_{a_1}$. Parents are motivated by altruism as described above.

As in Becker and Tomes (1986), optimal human capital accumulation is the same, if it is assumed instead that the child buys inputs to human capital accumulation in the market. This holds as long as (a) the bequest motive is operative, (b) the opportunity costs for inputs are the same as for the parent, and (c) credit markets are perfect. The opportunity cost of effective time for the parent is the after tax wage rate w_i, while that for goods (cx^*) depends on the tax treatment of goods inputs to child care and schooling.

Conveniently, this problem is separable: given (a_1, h_1) the problem of the independent agent is the same as described above and implies the value function $V(a_1, h_1)$. We can then write the problem of finding the optimal investment in h^* as

$$\max V(a_1, h_1)$$

$$\text{s.t. } h_1^* \text{ given; } h_1 = h^*_{T*+1}$$

(13) $$h^*_{i+1} = (1-\delta^*_h)\, h^*_i + G^*(v^*_i, x^*_i, \bar{h}_i)$$

$$a_1 = -\sum_{i=1}^{T*} (1+r)^{T*-i} \, (w v^*_i + p^* \, x^*_i)\,,$$

where v^*_i denotes *effective* time purchased in the market. The last constraint imposes that initial assets of a new household equal the cumulative value of expenditures on education prior to entering the model.

The marginal values of initial asset holdings and human capital endowments are

$$\partial V / \partial h_1 = w(1-l_1-v_1)\,\lambda / R_1 + [1-\delta_h + G_h(1)]\,\varphi_1 = \varphi_0$$

and $$\partial V / \partial a_1 = \lambda\,.$$

The first-order conditions for this problem are

(14) $$\varphi^*_i \, G^*_v(i) = \partial V \big/ \partial a_1 \, w(1+r)^{T*-i}$$

(15) $$\varphi^*_i \, G^*_x(i) = \partial V / \partial a_1 \, p^* \, (1+r)^{T*-i}$$

(16) $$\varphi^*_{T*} = \partial V / \partial h_1$$

(17) $$\varphi^*_{i-1} = (1-\delta^*_h)\,\varphi^*_i\,;\;\; i = 2, \ldots, T^*.$$

In addition, the aggregation conditions are modified as follows. Debt accumulated by households that have not yet separated from their parents must be subtracted from the capital stock. Given the flow budget constraint $a^*_{i+1} = (1+r)\,a^*_i - w\,v^*_i - p^*\,x^*_i$ and $a^*_1 = 0$, total debt hold by such households is

$$\sum\nolimits_{i=1}^{T*} a^*_i\,[(1+\gamma)\,(1+n)]^{T*-i}\,.$$

Similarly, labor supplied to firms is reduced by

$$\sum\nolimits_{i=1}^{T*} v^*_i\,[(1+\gamma)\,(1+n)]^{T*-i}\,.$$

The growth rate now obeys $(1+\gamma)^{t_b-1} = \varepsilon\,h_{t_h}\;/\;h^*_1$.

Assume a functional form for G^* analogous to that for G: $G^* = B^*\,(v^*)^{\psi^*}\,(x^*)^{\eta^*}\,\bar{h}^{\xi^*}$. Conveniently, given guesses for the marginal values of a_1 and h_1, this can be solved in closed form. The steps are

1. Compute φ^*_i recursively backwards from (17).

2. Solve (14) for the sequence $G^*_v(i)$.

3. Combine (14) and (15) into

(18) $$x^*_i \big/ v^*_i = (\eta^*\,/\,\psi^*)\,/(w\,/\,p^*)$$

and use it to solve for time input

$$(v^*_i)^{\alpha^*+\eta^*-1} = \frac{G^*_v(i)}{\psi^*\,B^*\,\bar{h}^{\xi}_i\,(\eta^*\,w\big/\psi^*\,p^*)^{\eta^*}}\,.$$

4. Recover goods inputs from (18).

5. Compute asset holdings from the flow budget constraint and h_1 from the law of motion (13).

The cost parameter p^* is computed analogously to p. Since goods inputs should be thought of as schooling expenditures of parents, none of the goods inputs are paid for by foregone earnings and none of these inputs are tax deductible. However, following Trostel (1993, p. 333), it is assumed that such inputs are subsidized at a rate of 60 percent. This is based on the assumption that goods inputs do not include child rearing costs. According to Trostel, the government pays for more than 80 percent of education costs and for 70 percent of high school and college expenses. A subsidy rate of 60 percent is therefore conservative.

2.4 Comparison With Infinite Horizon Models

In order to determine the nature of the differences between the OLG model presented here and the infinite horizon models studied in the literature, we develop a sequence of models that transform the OLG model into an IH model, changing one assumption at a time. The sequence of changes is chosen so as to maintain a reasonable OLG model as long as possible. The sequence is as follows:

1. Baseline OLG model.

2. Add altruistic bequest motive with $b = 1$.

3. Eliminate population growth.

4. Eliminate retirement.

5. Parents internalize the intergenerational human capital spillover.

6. Children are born after parents have died ($t^* = T+1$) and human capital spillover occurs only at that time.

7. Children inherit *all* of their parents human capital ($\varepsilon = 1$).

Up to step 4, the changes are either harmless or somewhat reasonable. Assuming that parents internalize the human capital spillover is certainly too strong and takes the notion of a spillover from parents to children too literally. The assumption that children inherit all of their parents' human capital implies that the model no longer replicates observed age-earnings profiles.

The final two steps introduce the most unreasonable assumptions and generate a standard infinite horizon model. To see this, note that with $b = 1$, $N_c = 1$ and $t_b = T + 1$, the objective function in (4) reduces to the infinite horizon version

$$\sum_{i=1}^{\infty} \beta^i u(c_i, l_i) .$$

Similarly, with $\varepsilon = 1$ and $t_h = T + 1$, we have $h_1^* = h_{T+1} = (1-\delta_H) h_T + G(T)$ and the law of motion for h holds over an infinite horizon:

$$h_{i+1} = (1-\delta_H) h_i + G(i) ; \quad i = 1, 2, \dots$$

Finally, with $t^* = T - t_b + 1 = 0$, parental bequests directly augment the capital endowment $(a_1^* = B_T)$ and the budget constraint (2) also holds over an infinite horizon.

3. Parameter Choices

3.1 Firms

Parameter choices for the production technology are conventional. The capital share is $\alpha = 0.3$ and the depreciation rate of physical capital is set to 0.05 as in many previous studies. The parameter A can be normalized to one. With a capital-output ratio of 2.5, these parameters imply an investment share of 20 percent $(I/Y = [\gamma + n + \delta_k] K/Y)$ and a pre-tax interest rate of 7 percent $(\alpha Y/K - \delta_k)$.

3.2 Government

The income tax rate chosen for this study is constructed from the IRS Individual Income Tax Returns for 1988. A simple calculation allows to place tight bounds on the average marginal tax rate. A lower bound is given by the overall average tax rate of 20 percent, computed as the ratio of total tax payments to adjusted gross income. An upper bound of 25 percent is provided by the maximum average tax rate paid by any income group. A more precise calculation shows that the U.S. tax system is only very slightly progressive. The average marginal tax rate, weighted by the size of tax payments of the different income groups, is only a little above 20 percent. To this, a 10 percent social security tax is added. Therefore, τ_w is set to 0.3.

The capital income tax is interpreted to encompass, in addition to personal income taxes, corporate profit taxes and a part of property taxes. The resulting tax rate of 37.5 percent is quite close to the more careful estimate in Kim (1992). The consumption tax rate is computed from Fullerton and Rogers (1993, table 3-6). None of depreciation is tax deductible ($\kappa = 0$). Government consumption is 18 percent of gross national product. The ratio of government debt to GDP is set to 0.37, the average ratio of government net financial assets to GDP during the period from 1960 to 1990. The level of transfers is adjusted to balance the government budget on the initial balanced growth path. For simplicity, we assume that individuals of all ages receive the same amount of transfers at any given date.

The main reason for including a social security tax is to capture its effect on work incentives past age 65. If tax rates are independent of age, households never retire. However, means testing of benefits introduces a 33 percent marginal income tax rate on most recipients. In addition, for some households with higher incomes social security benefits are subject to income taxation.[1] All of these provisions are loosely captured by a flat additional wage tax (τ_o) in the present model. Its revenues are assumed to be part of the overall government budget. The tax rate is set somewhat arbitrarily to 25 percent. For the purposes of this paper, there is no need to model social security benefits.

3.3 Households

3.3.1 Demographics

The size of the cohort born at the initial date is normalized to one. Subsequent generations grow at a rate of n = 1.24 percent per year, which matches the U.S. population growth rate. No attempt is made to account for lifetime uncertainty or to achieve a realistic age structure of the population. However, variations in labor force participation rates are captured through labor/leisure choice.

Households enter the model at age 20 (model age 1). They live until age 74 (T = 55) and retire at age 65 (T_1 = 46). More precisely, earnings are subject to additional taxation after age 65, which induces agents to partially retire endogenously. Children are born when parents are t_b = 28 years old and "inherit" a fraction (ε) of the human capital stock of the parental generation at parental age t_h = 38. In the model, children therefore enter at model age 28 of their parents (they are born at model age 8, but enter only 20 years later) and "inherit" human capital at parent's model age 18. The fraction of human capital "inherited" from the previous generation (ε) determines the growth rate of the economy. This is set match the annual growth rate of U.S. per capital gross national product between 1950 and 1992 (1.7 percent).

3.3.2 Preferences

The utility function is assumed to be of the form $u(c,l) = c^{1-\sigma}\, l^{\rho(1-\sigma)}/(1-\sigma)$. A sufficient condition for strict concavity is $\sigma > 1$. To see this, note that $u_{cc} = -\sigma c^{-(1+\sigma)}\, l^{\rho(1-\sigma)} < 0$, $u_{ll} = \rho[\rho(1-\sigma)-1]c^{1-\sigma}\, l^{\rho(1-\sigma)-2} < 0$, and $u_{cl} = \rho(1-\sigma)c^{-\sigma}\, l^{\rho(1-\sigma)-1}$. Strict concavity then requires $u_{cc}u_{ll} - (u_{cl})^2 > 0$ which simplifies to $\sigma[1-\rho(1-\sigma)] - \rho(1-\sigma)^2 > 0$ or $\sigma > \rho/(1+\rho)$.

[1] See Feldstein and Samwick (1992) for details and estimates of the implied marginal tax rates.

3.3.3 Intergenerational Links

The parameter b can be calibrated so as to replicate observed bequest flows. Using data from the 1986 Survey of Consumer Finances, Gale and Scholz (1994) estimate aggregate annual bequest flows of 0.88% of household net worth. In addition, there are intended inter vivos transfers of about the same size. This is broadly consistent with Kotlikoff and Summers (1981) who estimate transfer flows between 0.7 percent and 1.17 percent of net worth, depending on how broadly transfers are defined. Given that the bulk of bequests appears accidental (Menchik and David 1983), a target level of 1 percent of net worth (the aggregate capital stock in the model) is chosen. In the model, if the cohort born at date 1 leaves B_T, then the aggregate bequest flow at date 1 is $B_T\,[(1+n)(1+\gamma)]^{1-T}$. Household net worth is simply $K + D$.

3.4 Human Capital Accumulation

It is useful to begin with an explicit mapping from model variables into observables. In the baseline case, households enter the model at age 20. Most human capital accumulation should therefore be thought of as on-the-job training (OJT). The time input v_t corresponds to the fraction of time spent on OJT; the fraction of market time spent on training is therefore $v_t/(1-l_t)$. Goods inputs correspond any additional employer's costs for training. Hourly wages relative to age 20 are mapped into h_t/h_1. Market time in the model is $(1 - l_t)$ which corresponds in the data to the fraction of total time spent on work-related activities for the relevant age groups. Variations of market time with age reflect changes in hours worked as well as labor force participation rates. In all cases, it is important to account for differences between women and men.

The human capital production function takes the standard exponential form as in most of the human capital literature: $G(.) = B v^{\psi} h^{\zeta} x^{\eta} \bar{h}^{\xi} \tilde{h}^{\chi}$. The productivity parameter B is chosen so that wages increase by 62 percent between the ages of 20 and 45 (based on data described below). In the baseline case, there is no ability effect and $\chi = 0$. The remaining parameter choices combine results from econometric studies with information contained in the observed life-cycle profiles of wages, market time, education time and earnings. Heckman (1976) and Haley (1976) provide widely cited estimates of a human capital production function. In both studies, point estimates of the returns to private inputs are around $\eta + \psi = 0.55$ with confidence intervals ranging to approximately 0.7.[2] Balanced growth, on the other hand,

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[2] Point estimates in Haley (1976) are between 0.544 and 0.590 depending on education. The point estimate in Heckman (1976) is 0.52.

requires constant returns to reproducible factors: $\xi = 1 - \psi - \eta$. The implied spillover of $\xi = 0.45$ appears large; therefore we set ξ to the smallest value consistent with the estimated confidence interval for $\eta + \psi$ ($\xi = 0.3$). The vast literature on schooling also suggests that such spillovers are important without providing a reliable estimate though (Hanushek 1986). While there is reason for caution as empirical evidence in the area of human capital is generally controversial and fraught with data problems, the case for the presence of spillovers appears quite strong.

Following the common approach in the literature, it is assumed that time and human capital enter the production function as *effective time* ($v\ h$): $\zeta = \psi$. Mincer (1993, p. 270) estimates that half of job training costs are time inputs of the workers being trained. We therefore set $\eta = \psi$.[3] We therefore need $\eta = \psi = \zeta = 0.35$. If there are ability effects, the values for ζ and χ are simply exchanged.

Evidence on the depreciation rate of human capital is mixed. Mincer (1993, p. 269) estimates a rate of 4 percent per year from PSID data, but gives no standard errors. The point estimate in Heckman (1976, table 1) is 3.7 percent with a standard error of 1.5 percent. Haley (1976, table III) reports point estimates ranging from 1.7 to 4.3 percent depending on education.[4] Since these estimates are derived from cross-sectional data, they likely overstate the true rate of depreciation. As illustrated below, once it is taken into account that wages grow over time, empirical wage profiles show very little decline at old ages when training has largely ceased and wage growth is largely determined by depreciation. In order to replicate this observation, it is necessary to choose an low value of $\delta_h = 0.01$. Alternative values are explored in the sensitivity analysis.

Stokey and Rebelo (1995, p. 543) estimate that approximately 2.5 to 4 percent of human capital are lost each year due to retirement. Therefore, for infinite horizon models, they consider depreciation rates between 2.7 and 8 percent reasonable.

3.4.1 Investment in the Endowment

The length of the investment period is set to $T^* = 10$ years. Little is known about production functions for schooling, not to mention child rearing. It is therefore assumed that the

[3] An alternative approach that yields similar results is to calibrate ψ to match observed time spent on training.

[4] In infinite horizon models it is appropriate to choose larger values because some depreciation of human capital occurs through death.

functional form is identical to that for job training. The productivity parameter B^* equals B. Alternatively, to explore the impact of larger investments, it is chosen so that h^* increases by 50 percent over the investment period.

For η^*, the share of goods inputs in training, some direct evidence exists. For schooling, Johnson (1978) estimates that the ratio of factor shares (ψ^*/η^*) is around six. Becker (1975) estimates the share of goods in private education costs to be around 0.25. As usual in the area of human capital, the estimates should be viewed with some caution, but the consensus view in the literature seems to be that foregone earnings constitute the most important cost of education. We therefore set ψ^*/η^* to a value near 2. To obtain the same returns to scale coefficient, we need $\psi^* + \eta^* = 0.7$. Therefore, $\psi^* = 0.46$ and $\eta^* = 0.24$.

3.4.2 Age-Earnings Profiles

Largely due to the lack of longitudinal data, little consensus exists as to the exact shape of age-earnings or age-wage profiles. Frequently, a quadratic function is fitted to a cross-sectional profile constructed for white men. However, it is generally recognized that (a) the quadratic is a poor approximation (Murphy and Welch 1990); (b) longitudinal profiles are likely steeper because they reflect wage growth over time; (c) the profiles for women and minorities differ substantially from those for white men.

To illustrate the differences, using cross-sectional data, Murphy and Welch (1990) find that weekly earnings for white men roughly double after 30 years of experience. In contrast, the often used data of Welch (1979) only show wage growth of 50 percent.

Wage profiles for women and minorities are substantially flatter. Duncan and Hoffman (1979) find wage growth of 71% for men, but only 43% for women after 30 years of experience. The differences are even larger in Polachek and Siebert (1993, p. 3): average weekly earnings rise by 87 percent for men, but only by 34% for women (these are somewhat larger growth factors because the initial age group is 16–24 years instead of the usual 20–25 years). For the UK, the corresponding numbers are 61 percent for men and 22 percent for women.

Given that real wage growth in the U.S. has been close to zero for decades, the baseline calibration follows the bulk of the literature in assuming that the cross-sectional wage profile is the same as the longitudinal one. The sensitivity analysis explores a wage profile with a one percent per year growth adjustment. The target age-wage profile is calculated as a weighted average of the male and female wage growth factors. Using data from Polachek and Siebert (1993, p. 3) and a fraction of white male workers of 0.56 (Duncan and Hoffman 1979) we obtain Table 1. Profiles constructed from longitudinal data (synthetic cohorts; see Owen 1986; Juster and Stafford 1985) are broadly similar to the growth adjusted profiles constructed here.

Data for market time are taken from Fullerton and Rogers (1993, table 3-7). These include age-dependent changes in hours worked as well as labor force participation for male and female workers. Multiplying market time and wages yields earnings, where it is assumed implicitly that market time profiles are identical for men and women.

In the baseline case, B is chosen to replicate earnings growth between the ages of 20 and 45. However, for some extreme parameter values, the model predicts zero earnings and wages for several years at the beginning of life. This problem only arises, if the OLG model is computed with preference and human capital technology parameters taken from the IH model (especially a high depreciation rate of human capital). In that case, parameters are chosen to match the growth rate of human capital implied by observed earnings and time allocation instead.

3.4.3 On-the-job Training

Time-use data are available for calibrating the age profile of v_t (Juster and Stafford 1985). Mincer (1993, chapter 9) uses this data to construct overall measures of OJT costs.[5] For men, training time declines roughly linearly from 9.7 hours per week at ages less than 25 years to zero at age 65. Two adjustments must be made. First, most job training is joint with production; only part of this should be included in v_t. Secondly, women receive much less training than men.

The target profile for this paper is constructed as follows (see Table 2). Total hours of training are computed as in Mincer, except that half of the time spent on training joint with production is counted (instead of one-third, which Mincer calls conservative). Assuming that market time for this sample of workers is 40 hours per week yields the fraction of market time spent on training for men. As in Mincer, I assume that women receive half of that amount and that men constitute 56 percent of workers at all ages (Duncan and Hoffman 1979). This yields the average fraction of time spent on training for men and women in the last column of the table.

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[5] The results of Rosen (1982), while estimated quite differently, are similar.

4. Model Evaluation

To evaluate model performance, it is compared with a number of additional observations described in this section.

4.1 Age Profiles

The empirical age-wealth profile is taken from Huggett (1996, p. 487). Each data point represents mean wealth holdings of an age group. Wealth includes housing, but not social security wealth. Because of the enormous heterogeneity in the data, it is difficult to compare levels. However, the profiles of wealth holdings relative to the peak at age 60 are very similar for mean and median holdings.

4.2 Aggregates

Target values for most aggregates are conventional. Given a capital-output ratio of 2.5 and a depreciation rate of 0.05, an investment share of 0.2 (0.074 for net investment) is implied by the balanced growth restrictions. Since the share of government spending in GNP is 0.18, the consumption share must be 0.62. Care must be taken because observed output does not include the value of human capital produced. Model GNP therefore corresponds to $Y - X$.

5. Program Files

5.1 Procedures

5.1.1 Startup/Initialization

It is necessary to create the directories defined in og1ini. It is possible to change directory names by simply redefining the globals in og1ini. After starting Matlab, switch to the program directory and initialize globals with og1ini. This temporarily modifies the Matlab path.

5.1.2 Creating a New Experiment

This requires several steps.

(1) Choose a new calibration number. Define the calibration in cal_set.

(2) Run cal_set and then parasave to save the calibration.

(3) Choose a new bgpNo. Define the policy parameters in bsetting.

(4) Create a file with initial guesses. This is best done by copying an existing result file (b*.mat). If that is not possible, create a new file with bgp_guess. Without good initial guesses, it is often difficult to achieve convergence. Then try to set *B* to a low value initially; this converges more reliably.

(5) Define target values for calibrated parameters in cal_set.

(6) Run the calibration, calibr.

(7) To run another balanced growth path for the same parameter choices: Create a file with initial guesses and use runbgp.

If the algorithm fails to converge, it often helps to use simpler problems as initial guesses. In particular, it may help to (a) reduce the bequest parameter, (b) set hcAltruism = 0, (c) raise σ, (d) increase the number of iterations in hhprob.

5.1.3 Computing a Calibration

Assuming that the experiment has been defined (see above), simply run calibr.

5.1.4 Computing a Balanced Growth Path

The actual routine that computes the balanced growth path is og1bgp, but a more convenient interface is provided in runbgp. This requires a file containing an initial guess of appropriate dimension. If no such file is available, see above instructions for creating an experiment.

bgpcmp compares results of two balanced growth paths. To show just one path, use bgp_show.

5.1.5 Checking Correctness of the Computations

The code that implements the computations is too complicated to be checked directly. Therefore, a number of routines are provided that directly implement the equilibrium conditions described in Section 2. All conditions are checked automatically during the computation. Individual routines are listed in the following table.

bgpchk	Checks all equilibrium conditions, except for those of the household problem
hhdev	Deviations from household optimality conditions, except investment in h_1
hhdev_h1	Deviations from optimality conditions for investment in h_1

5.2 Solution Method

The basic algorithm is similar to that of Auerbach and Kotlikoff (1987). The main complication is the possibility of binding nonnegativity constraints on work time and inputs to training. Since the large number of states rules out most computational methods, two nested Gauss-Seidel loops are employed. The outer loop iterates over guesses of aggregates such as the capital stock, tax rates, etc. The inner loop solves the household problem. The inequality constraints are handled by explicitly imposing the Kuhn-Tucker conditions and iterating over the corresponding multipliers.

Solving the model with bequests is more difficult. Convergence is achieved more easily, if the following techniques are followed:

- Allow sufficient numbers of iterations in hhprob2 to ensure that the household problem is solved to high precision all the time.
- Adjust shadow prices of children (value of bequests and human capital spillovers) slowly. Otherwise *h* oscillates.

5.3 Calibration

M-File	Purpose
cal_fn	file name for calibration results
cal_set	set exogenous parameters
Calibr	calibrate parameters from initial bgp
Matchval	compute values to be matched by bgp/calibration
Paraload	load calibrated parameters
ParaSave	save calibrated parameters
Parashow	show calibrated parameters

5.4 Computing Balanced Growth Paths

M-File	Purpose
aggreg	aggregate individual -> aggregate factor supplies
aggreg2	aggregate individual -> aggr. stocks; for variables during h1 investment phase
aggregX	aggregate individual -> aggregate stocks
bgp_guess	create a file with initial guesses
bgpBequ	bequest data for bgp
bgpChar	characterize bgp by summary statistics (for calibration)
bgpChk	Check bgp for consistency
BgpCmp	Compare two bgps
bgpfname	bgp file name
bgpload	load bgp results
bgpPol	construct true policies for bgp
bgppolcy	policy compatible with a particular bgp
bgprw	factor prices and labor tax facing hh
bgpsave	save bgp results
bgp_show	show summary statistics for bgp
bgpStats	summary statistics of bgp
bgpStore	Show path of several variables over iterations of og1bgp. Mainly a debugging device.
BSetting	Settings for bgp experiment
og1bgp	compute bgp
runbgp	simplified interface for og1bgp
setBgPol	set policy guess in bgp file

5.5 Household Problem

M-file	Purpose
h1_invest	Optimal investment in h_1
hhAssets	Compute asset profile and terminal savings
hhbc	Compute present value of expenditure and income
hhdev	HH deviations
hhdev_h1	Deviations from household FOCs for investment in h1.
hhprob	HH problem; solution given w, r and checking results
hhprob2	HH problem; ignore non-negativity on bequests (if bequ > 0)
hhprob3	HH problem; h exogenous
hhiter	One iteration; produces solution given w, r, phi, lambda
hhSens2	Sensitivity of hh problem; changes in (w,r)
hhSensit	Sensitivity analysis of hh problem
hhshow	Show solution to hh problem
netEarn	Earnings net of labor taxes

5.6 Government

M-file	Purpose
GovBC	government bc and components
GovDefic	government budget deficit
ktax	capital income tax revenue
ktaxBase	capital income tax base
ltaxBase	labor tax base contribution of cross section of households
ltxBase2	labor tax base for series of dates
ltax	labor income tax revenue; X tax deductible
ltaxhh	contribution to labor tax base of hh
PolChk	check consistency of policy paths
PolComp	compare policy path with specification in expmt
polname	name of policy variable to adjust
taxrev	total tax revenue
taxvec	break up vectors (tax0) into individual tax rates
x_cost	cost of x to household, given policies

5.7 Functional Forms

Calculations that depend on functional forms are collected, as far as possible and efficient, in the following files.

M-file	Purpose
Gf	human capital production function and derivatives
prodFct	production function for goods
Ufct	utility function and derivatives
ULife	lifetime utility
UtilDiff	welfare gain from change of utility
kBounds	find K/L s.t. interest rate is bounded in a given interval

5.8 General Purpose Functions

M-file	Purpose
demogr	Demography facts
grRate	Growth rate
hhFacts	Facts about a hh born at a given date
hPrLoad	Load h profile from a bgp file
ParaChk	Check that calibrated parameters satisfy consistency cond.
PdAvg	Averages over annual periods
PdToYr	Convert a vector x(period) -> x(year)
PopSize	Population size

5.9 Notation in the Program Code

The following table provides a mapping from the notation used in the text to that in the program code (which obviously does not allow Greek letters). Trivial cases are omitted. The human capital production function is written

G = B .* h.^ksi .* v.^psi .* x.^eta .* hAvg.^zeta .* z.^gg.

Households	Firms	Human capital production
σ = sig; ρ = rho; β = bb ε = hcInherit; b = bequ	α = aa; δ_K = ddk	η = eta; ψ = psi; ζ = ksi ξ = zeta; χ = gg; δ_h = ddh
Demographics	Government	Investment in h_1
$1 + n$ = popGrowth T_1 = T1	τ_k = tk; τ_w = tw; τ_c = tc τ_O = tOld; τ_s = ts; Tr/Y = TrY G/Y = GY; D/Y = DY Λ = xEarn; Γ = xDeduct	η^* = etax; ψ^* = psix; ζ^* = ksix ξ^* = zetax; δ_h^* = ddhx; a_1 = age1x; h_1^* = h1x;

5.9.1 Switches

A number of switches determine the type of experiment to be computed. A value of 1 typically indicates “true,” while 0 indicates “false.”

Switch	Definition
dbg	Debugging switch; determines the amount of error checking performed during calculation; set to low value (0 or 1) to achieve high speed; set to high value (10 or 100) to achieve maximum error checking
DoCalib	Are we calibrating or computing a balanced growth path for given parameters?
hEndog	Is human capital endogenous? (Always 1 for this paper.)
h1Invest	Is there investment in the endowment?
hAbility	Are there ability effects in human capital accumulation?
hcAltruism	Do parents internalize the human capital spillover?
hSpill	Type of intergenerational human capital spillover
.type	1 = spillover at a point in time; 2 = spillover over a number of periods
.age1, .age2	range of ages over which spillover occurs (if hSpill.type = 2)
calH	Calibration of human capital
.calB	1: B matches wage growth; hcInherit matches growth rate (baseline) 2: B matches growth rate; hcInherit exogenous
.hcInherit	Value for hcInherit, if exogenous
calBeta	Calibration of β
.calType	0: β exogenous; 1: β matches *K/Y*; 2: β exogenous (slow adjustment)
.bbTarget	Target value, if β exogenous
.KYTarget	Target value for *K/Y*
bequTarget	Calibration of *b*
.calBequ	0: *b* exogenous; 1: *b* moves slowly towards exogenous target; 2: *b* matches bequest flow / GDP
.bequ	Value for *b* if exogenous
.flow	Target value for bequest flow / GDP

5.9.2 Additional Variable Definitions

Variable	Definition
a1	Asset holdings at birth. Household earns interest on this in the first period and effectively starts out with a1 * (1+r(1)).
age1x	Age at which investment in h_1 begins; no investment in h_1, if age1x > iniAge
BxTarget	Structure with calibration target for Bx. Elements:
	.hGrowth target growth factor for h* over investment period
grTarget	Target growth factor for per capita income. Used to calibrate hcInherit.
hcAge	Model age of parents at which human capital is transferred
hcYear	Physical age corresponding to hcAge
hExog(1:T1)	Human capital profile; only for experiments with exogenous human capital
hSpill	Structure. Data about human capital spillover. .Type: type of spillover; 1: spillover at one age only; 2: spillover over range of ages .age1, .age2: age range over which spillover occurs (if .Type = 2)
hTarget	Structure. Target for calibration of B. age1 and age2 determine the ages over which wage growth is computed. hGrowth is the target growth factor.
iniAge	Age at which household enters the model
iniPop	Size of cohort born at date 1
IYTarget	Target investment share. Used for calibrating δ_K. Not used for baseline calibration.
lTarget	Target for leisure share in total time. Used to calibrate ρ.
tbYear	Physical age when parents have children
Tx	Length of time during which investment in h1 takes place
yPerPd	Number of years per period (usually 1); must equal yearsOfLife / T

To clarify the logic of the demographic variables in the code:

- A household lives for (yearsOfLife + iniAge) years. Physical ages are iniAge through (iniAge + yearsOfLife – 1), e.g. 20 through 74.
- Households have children at physical age tbYear, e.g. 28. These enter the model tbYear years after their parents.
- Children start out with a fraction of the human capital of their parents when these are hcYear years old. That is in period hcAge = hcYear – iniAge + 1 of the parent's life. E.g., if children inherit human capital at age 10, then hcYear = tbYear + 10 = 38 and hcAge = 18.

To convert physical age into model age, use the formula

$$\text{modelAge} = 1 + (\text{physicalAge} - \text{iniAge}) / \text{yPerPd}.$$

Conversely, to convert model age into physical age use

physicalAge = ([modelAge – 1] * yPerPd) + iniAge.

5.10 Individual Routines

5.10.1 hhprob

Updating of the bequest left:

Take the bequest received and the marginal utility of the child as given. Compare marginal utilities of child and parent. Adjust the bequest until both are equal. A more efficient method might be to compute c_T directly from c_{t*} of the child, back out the entire sequence of c_t's from the Euler equation and determine the bequest left residually.

6. Experiments

An experiment consists of (1) calibrating a balanced growth path for a set of initial tax policies; (2) computing an alternative balanced growth path with the same parameters but different tax policies. It therefore involves three numbers: (1) calNo indicates the parameter choices and calibration targets (see cal_set); (2) calBgp indicates the policy choices of the initial bgp; (3) bgpNo indicates policy choices for the second bgp, as defined in bsetting. Results are stored in two types of files:

Calibration Files. These contain parameters that were either set exogenously or calibrated (using calibr) to match certain observations (see above). File names (c000_000_000.mat) are determined by cal_fn and stored in the directory defined by calDir. The detailed structure of the file is documented in the program that creates it (parasave). Settings for calibrations are defined in cal_set.

Balanced Growth Result Files. These contain the results of an experiment, that is, of a computed balanced growth path. File names are determined by bgpfname and stored in the directory defined by bgpDir. The detailed structure of the file is documented in the program that creates it (bgpsave). Settings for balanced growth paths are defined in bsetting. These are mostly policy parameters such as tax rates.

The experiments reported in Tables 5 and 6 in the main text can be computed in one step with bgp_table. This also maps experiment numbers (calNo, calBgp, bgpNo) as used in cal_set and bsetting into entries in those tables.

Results for a single balanced growth path are shown with bgp_show, which also produces the model evaluations of Figure 1 and Table 2. Table 4 and Figures 2 and 3 are produced with bgpcmp, which compares two balanced growth paths.

7. Growth Effects With Infinite Horizons

With infinite horizons, it is possible to show analytically that (*i*) the tax treatment of depreciation alters growth effects in important ways (see also Stokey and Rebelo 1995), and (*ii*) diminishing returns imply smaller growth effects.

7.1 Tax Treatment of Depreciation

To see the importance of the tax treatment of depreciation consider the following tractable special case.[6] A representative household solves

$$\max \int_0^\infty e^{-\rho t} \frac{c_t^{1-\sigma}}{1-\sigma} dt$$

subject to the accumulation conditions for physical and human capital

$$\dot{k}_t = I_{1t} - \delta_1 k_t$$

(19)
$$\dot{h}_t = I_{2t} - \delta_2 h_t .$$

and the budget constraint

$$p_{1t} I_{1t} + p_{2t} I_{2t} + c_t - q_{1t} k_t - q_{2t} h_t - T_t = 0 .$$

There are three sectors producing physical capital, human capital and final goods at tax inclusive prices p_{1t}, p_{2t}, and 1, respectively. I_{st} is investment in accumulation of factor *s*. The corresponding after-tax rental prices are q_{st}. T_t are lump-sum transfers. The stocks of human and physical capital are denoted by *k* and *h*, while their respective depreciation rates are δ_k and δ_h.

Assume that the production functions exhibit constant returns in (*k*, *h*) and denote them by

$$I_k = G(k_k, h_k)$$

$$I_h = H(k_h, h_h)$$

It is easy to show that a balanced growth path is characterized by

(20)
$$\frac{G_h}{G_k} = \frac{H_h}{H_k}$$

[6] This follows Stokey and Rebelo (1995).

where subscripts denote partial derivatives. In addition, the rates of return of both types of capital must equal the interest rate. In the case of taxation gross of depreciation:

$$(21) \qquad \begin{aligned} (1-\tau_1)\,G_k - \delta_1 &= r \\ (1-\tau_2)\,H_h - \delta_2 &= r \end{aligned}$$

Here τ_s is the tax rate on factor income in sector *s*. The assumption of a representative agent induces two peculiarities. First, the problem is indistinguishable from a standard two-capital good economy. The only difference between *k* and *h* is the label. Secondly, the production side alone, via (20) and (21), determines the factor input ratios in both sectors and the interest rate. Preferences translate this interest rate into the growth rate.

In the simplest symmetric case all income is taxed at the common rate $\tau = \tau_1 = \tau_2$ and depreciation rates are equal for both factors. Then input ratios are unaffected by taxes and the change in the interest rate is $\Delta r = -\Delta\tau\, G_k$ which implies a change in the growth rate of

$$(22) \qquad \Delta\gamma = \frac{\Delta r}{\sigma} = -\frac{\Delta\tau\, G_k}{\sigma} = -\Delta\tau\left(\gamma + \frac{\delta+\rho}{\sigma}\right).$$

If, on the other hand, returns are taxed net of depreciation, (21) becomes

$$(23) \qquad \begin{aligned} (1-\tau_1)[G_k - \delta_1] &= r \\ (1-\tau_2)[H_h - \delta_2] &= r \end{aligned}$$

such that the change in the growth rate is

$$(24) \qquad \Delta\gamma = \frac{\Delta r}{\sigma} = -\frac{\Delta\tau\,(G_k - \delta)}{\sigma} = -\Delta\tau\left(\gamma + \frac{\rho}{\sigma}\right).$$

Comparing (22) with (24) reveals that growth effects must be sensitive to the tax treatment and magnitude of depreciation. The difference in the change in the growth rate between the two cases is $\Delta\tau\delta/\sigma$ which is of the same magnitude as $\Delta\tau\gamma$, if the depreciation rate is around 0.1 as in the case of physical capital. However, the depreciation rate of human capital is much more controversial. Estimates used in the literature range from near zero to as much as 12 percent (Lord 1989). But even if the true value of the depreciation rate were known with certainty, it would be difficult to determine the appropriate value for the corresponding model parameter δ_h from it. For, in an infinite horizon model, δ_h has to capture obsolescence of knowledge as well as the depreciation of human capital at death. In general, it is not obvious how estimates of parameters in the data translate into parameter values in the reduced form model.

7.2 Diminishing Returns in an Infinite Horizon Model

This appendix demonstrates that factor taxation affects human and non-human wealth asymmetrically in the presence of diminishing returns to private inputs in human capital accumulation. In particular, if returns to scale are sufficiently small, the growth rate and the after-tax interest rate are left almost unchanged as the household adjusts physical capital much more than human capital.

Consider the same problem as in the previous section, except that human capital is accumulated subject to diminishing returns to private inputs:

$$\dot{h}_t = I_{2t}^{\varphi}\, \bar{h}_t^{1-\varphi} - \delta_2\, h_t \,.$$

Here, $\bar{h}$ denotes average human capital in the economy, which the household takes as given. Denote the co-states for the accumulation and budget constraints by $\lambda_{kt}, \lambda_{ht}, \mu_t$. Then the first-order conditions for maximization of the Hamiltonian are

$$c^{-\sigma} = \mu$$

$$\lambda_k = p_k\, \mu$$

$$\lambda_h = p_h\, \mu \left(I_h / \bar{h}\right)^{1-\varphi} / \varphi$$

$$\hat{\lambda}_k = \rho + \delta - q_k\, \mu / \lambda_k$$

$$\hat{\lambda}_h = \rho + \delta - q_h\, \mu / \lambda_h$$

The first-order conditions for firms producing investment goods are

$$\bar{q}_k = p_k\, G_k(z_k) = p_h\, H_k(z_h)$$

$$\bar{q}_h = p_k\, G_h(z_k) = p_h\, H_h(z_h)\,,$$

where $z_i \equiv h_i / k_i ; i = h, k$. Further, along a balanced growth path

$$\hat{c} = \frac{r-\rho}{\sigma} = \hat{h} = \left(I_h / h\right)^{\varphi} - \delta,$$

where the after-tax rate of return is determined by $r = q_k / p_k - \delta$. If the production functions are assumed to be Cobb-Douglas, that is

$$G(k,h) = B_k \, k^{\alpha} \, h^{1-\alpha}$$
$$H(k,h) = B_h \, k^{\eta} \, h^{1-\eta}$$

then z_h is proportional to z_k and equilibrium can be characterized by a single equation in z_k.

$$\frac{(1-\tau)\, B_k \, \alpha \, z_k^{1-\alpha} - \delta - \rho}{\sigma} + \delta - \left(C_1 \, \varphi\right)^{\varphi/(1-\varphi)} \, z_k^{C_2} \, ,$$

where C_1 is a positive constant and $C_2 \equiv -\frac{\varphi}{1-\varphi}(1-\alpha+\eta) \leq 0$.

It follows that the elasticity of z_k with respect to $1-\tau$ is

$$E(z_k, 1-\tau) = -\left(1-\alpha-C_2\left[1+\left(\delta-\frac{\rho+\delta}{\sigma}\right)\frac{\sigma}{(1-\tau)\, B_k \, \alpha \, z_k^{1-\alpha}}\right]\right)^{-1} .$$

For $\varphi \to 0$, $E(z_k, 1-\tau) \to -1/(1-\alpha)$, such that $(1-\tau) z_k^{1-\alpha}$ is constant. Thus, the after-tax rate of return and, by the Euler-equation, the growth rate are not affected by the tax. On the other hand, when $\varphi \to 1, C_2 \to -\infty$ and the elasticity approaches zero – this is the case of constant returns to private inputs typically analyzed in the literature where the pre-tax interest rate remains unchanged and the growth rate falls as discussed in the text.

Numerical examples show that the elasticity $E(z_k, 1-\tau)$ depends inversely on φ. That is, the lower the returns to private inputs, the smaller the long-run growth effects of taxation. The literature just happens to concentrate on the extreme case where growth effects are largest.

8. Tables

Table 1. Age-Wage Profiles

Age	Market Time	Cross-sectional data Male	Female	Average	Growth-adjusted data Male	Female	Average
16-24	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25-34	1.11	1.58	1.35	1.47	1.74	1.49	1.63
35-44	1.17	1.87	1.34	1.63	2.28	1.64	1.99
45-54	1.16	1.85	1.31	1.61	2.50	1.76	2.17
55-64	0.79	1.77	1.27	1.55	2.64	1.89	2.31

Source: Polachek and Siebert (1993); Duncan and Hoffman (1979).

Table 2. Job-Training Data

Age	Fraction with training	Hours of training: Separate	Hours of training: Joint with production	Total hours of training	OJT / market time
20	0.76	3.2	9.5	6.0	0.15
30	0.72	1.8	7.5	4.0	0.10
40	0.58	1.7	6.4	2.8	0.07
50	0.48	1.3	2.1	1.1	0.03
60	0.29	0.4	2.6	0.5	0.01

Source: Mincer (1993).

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